

**Math 115**  
**Spring 2019**  
**Lecture 23**

?  $a^2 + b^2 = c^2$  ?  
 $y = mx + b$  ?  $d = rt$

Factor Completely:

$$\begin{aligned} \textcircled{1} \quad x^3 - 9x &= x(x^2 - 9) \\ &= x(x^2 - 3^2) \\ &= \boxed{x(x+3)(x-3)} \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad 4x^5 - 108x^2 &= 4x^2(x^3 - 27) \\ &= 4x^2(x^3 - 3^3) \\ &= \boxed{4x^2(x-3)(x^2+3x+9)} \end{aligned}$$

Always take care of GCF.

$$\begin{aligned} \textcircled{2} \quad 2x^4 + 16x &= 2x(x^3 + 8) \\ &= 2x(x^3 + 2^3) \\ &= \boxed{2x(x+2)(x^2-2x+4)} \end{aligned}$$

$$\begin{aligned} \textcircled{4} \quad 2x^3 - x^2 + 8x - 4 &= \underbrace{x^2(2x-1)} + 4\underbrace{(2x-1)} \\ &= \boxed{(2x-1)(x^2+4)} \end{aligned}$$

$$\textcircled{5} \quad x^3 - 10x^2 + 9x$$

$$= x(x^2 - 10x + 9)$$

$$= \boxed{x(x-1)(x-9)}$$

$$\textcircled{6} \quad 5x^4 + 20x^3 - 160x^2$$

$$= 5x^2(x^2 + 4x - 32)$$

$$= \boxed{5x^2(x+8)(x-4)}$$

$$\textcircled{7} \quad x^2y + 19xy + 60y$$

$$= y(x^2 + 19x + 60)$$

$$= \boxed{y(x+4)(x+15)}$$

$$\textcircled{8} \quad x^2 - 10 + 4x$$

$$= x^2 + 4x - 10$$

$$= (x + \bullet)(x - \bullet)$$

1, 10

2, 5

Prime

$$\textcircled{9} \quad x^2(x+5) - 25(x+5)$$

$$= (x+5)(x^2 - 25)$$

$$= (x+5)(x^2 - 5^2)$$

$$= (x+5)(x+5)(x-5)$$

$$= \boxed{(x+5)^2(x-5)}$$

$$\textcircled{10} \quad x^3(x-4) - 64(x-4)$$

$$= (x-4)(x^3 - 64)$$

$$= (x-4)(x^3 - 4^3)$$

$$= (x-4)(x-4)(x^2 + 4x + 16)$$

$$= \boxed{(x-4)^2(x^2 + 4x + 16)}$$

$$\textcircled{12}$$

$$x^2(x+3) + 6x(x+3) + 9(x+3)$$

$$= (x+3)(x^2 + 6x + 9)$$

$$= (x+3)(x+3)(x+3)$$

$$= (x+3)^3$$

$$\textcircled{11} \quad 16x^2(4x-3) - 9(4x-3)$$

$$= (4x-3)(16x^2 - 9)$$

$$= (4x-3)(4x-3)(4x+3)$$

$$= \boxed{(4x-3)^2(4x+3)}$$

Find the missing factor:

$$(13) \quad 5x^2 + 22x + 8 = (5x + 2)(\underbrace{x + 4}_{\substack{2x \\ 20x}})$$

$$(14) \quad 4y^2 - 20y + 25 = (2y - 5)(\underbrace{2y - 5}_{\substack{-10y \\ -10y}}) = (2y - 5)^2$$

$$(15) \quad 6x^2 + 11x - 10 = (2x + 5)(\underbrace{3x - 2}_{15x})$$

$$(16) \quad 9x^2 - 49 = (\underbrace{3x + 7}_{-21x})(3x - 7)$$

$$(17) \quad 2x^2 + 13x + 15$$

$P=30$   
 $S=13$   
 $3, 10$

$$= 2x^2 + 3x + 10x + 15$$

$$= x(2x+3) + 5(2x+3)$$

$$= (2x+3)(x+5)$$

$$(18) \quad 12x^2 + 17x + 5$$

$P=60$   
 $S=17$   
 $5, 12$

$$= 12x^2 + 12x + 5x + 5$$

$$= 12x(x+1) + 5(x+1)$$

$$= (x+1)(12x+5)$$

$$(20) \quad 3x^2 + 5 + 20x$$

$$= 3x^2 + 20x + 5$$

$P=15$   
 $S=20$   
 $15$

**Prime**

$$(19) \quad 7x + 12x^2 - 12$$

$$= 12x^2 + 7x - 12$$

$P=-144$   
 $S=7$   
 $16, -9$

$$= 12x^2 + 16x - 9x - 12$$

$$= 4x(3x+4) - 3(3x+4)$$

$$= (3x+4)(4x-3)$$

(21)  $8x^3 - 17x^2 + 9x = x(8x^2 - 17x + 9)$   
 $\underbrace{8x^2 - 8x}_{8x(x-1)} \underbrace{-9x + 9}_{-9(x-1)}$   
 $= (x-1)(8x-9)$   $x(x-1)(8x-9)$

$P=72$   
 $S=-17$   
 $-8 \& -9$

(22)  $15x^2y^2 - 16xy^2 - 15y^2$   
 $= y^2(15x^2 - 16x - 15)$

$P=-225$   
 $S=-16$   
 $-25 \& 9$

$y^2(3x-5)(5x+3)$

$\underbrace{15x^2 - 25x}_{5x(3x-5)} \underbrace{+9x - 15}_{+3(3x-5)}$   
 $-225$

(23)  $9x^2 - 24x + 16 = 9x^2 - 12x - 12x + 16$   
 $P=144$   
 $S=-24$   
 $-12 \& -12$

$144$

$= 3x(3x-4) - 4(3x-4)$   
 $= (3x-4)(3x-4) = \boxed{(3x-4)^2}$

(24)  $16x^2 + 24x + 9$   
 $P=144$   
 $S=24$   
 $12, 12$

$144$

$= 16x^2 + 12x + 12x + 9$   
 $= 4x(4x+3) + 3(4x+3)$   
 $= (4x+3)(4x+3)$   
 $= \boxed{(4x+3)^2}$

Use Zero-Factor Prop. to Solve:

$$\textcircled{1} (3x-5)(x+8)=0$$

$$3x-5=0 \quad \text{OR} \quad x+8=0$$

$$x = \frac{5}{3}$$

$$x = -8$$

$$\{-8, \frac{5}{3}\}$$

$$\textcircled{2} x(2x-7)=0$$

$$x=0 \quad \text{OR} \quad 2x-7=0$$

$$x=0 \quad x = \frac{7}{2}$$

$$\{0, \frac{7}{2}\}$$

$$\textcircled{3} (x+4)(x-4)(x-10)=0$$

$$x+4=0 \quad \text{OR} \quad x-4=0 \quad \text{OR}$$

$$x=-4$$

$$x=4$$

$$\text{OR} \quad x-10=0$$

$$x=10$$

$$\{\pm 4, 10\}$$

Solve by factoring:

$$x^2 - 22 = 9x$$

$$x^2 - 22 - 9x = 0$$

$$x^2 - 9x - 22 = 0$$

$$(x-11)(x+2)=0$$

by Z.F.P.

$$x-11=0$$

$$x=11$$

$$x+2=0$$

$$x=-2$$

$$\{-2, 11\}$$

$$4x^2 = 20x - 25$$

$$4x^2 - 20x + 25 = 0$$

$$\begin{array}{c} \swarrow \quad \uparrow \quad \searrow \\ \quad \quad 100 \end{array}$$

$$P=100$$

$$S=-20$$

$$-10, -10$$

$$4x^2 - 10x - 10x + 25 = 0$$

$$2x(2x-5) - 5(2x-5) = 0$$

$$(2x-5)(2x-5) = 0$$

$$2x-5=0 \quad x = \frac{5}{2}$$

$$\{\frac{5}{2}\} \text{ Repeated Soln.}$$

Find two cons. odd integers such that their product is 15.

$$\rightarrow x \text{ \& \# x+2}$$

$$x(x+2) = 15$$

$$x^2 + 2x = 15$$

$$x^2 + 2x - 15 = 0$$

$$(x+5)(x-3) = 0$$

by Z.F.P.

$$x+5=0 \quad \text{OR} \quad x-3=0$$

$$x = -5$$

$$x = 3$$

$$3 \text{ \& \# } 5$$

OR

$$-5 \text{ \& \# } -3$$

Find two cons. even integers such that the sum of their squares is 100.

$$x^2 + (x+2)^2 = 100$$

$$x \text{ \& \# } x+2$$

$$x^2 + (x+2)(x+2) = 100$$

$$x^2 + x^2 + 2x + 2x + 4 - 100 = 0$$

$$2x^2 + 4x - 96 = 0$$

Divide by 2 to reduce

$$x^2 + 2x - 48 = 0$$

$$(x+8)(x-6) = 0$$

$$\rightarrow x+8=0 \quad \text{OR} \quad x-6=0$$

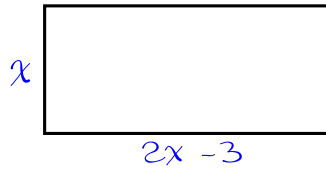
$$x = -8$$

$$x = 6$$

$x$	$x+2$
6	8
-8	-6

Area of a rectangular room is  $35 \text{ m}^2$ .

The length is 3m shorter than twice the width. Find its dimensions.



$$A = 35$$

$$LW = 35$$

$$x(2x-3) = 35$$

$$2x^2 - 3x - 35 = 0$$

$$2x^2 + 7x - 10x - 35 = 0$$

$$x(2x+7) - 5(2x+7) = 0$$

$$(2x+7)(x-5) = 0$$

$$x = -\frac{7}{2}$$

$$x = 5$$



5m by 7m

$$P = -70$$

$$S = -3$$

$$7 \times -10$$

The graph of  $y = x^2 - 10x + 24$  has two  $x$ -Ints. Find both of them.

$$y = 0 \rightarrow x^2 - 10x + 24 = 0$$

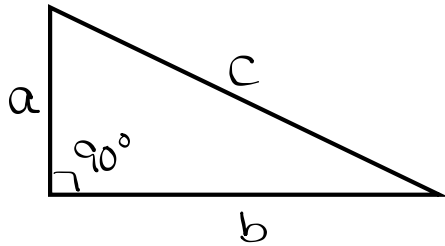
$$(x-6)(x-4) = 0$$

$$x = 6$$

$$x = 4$$

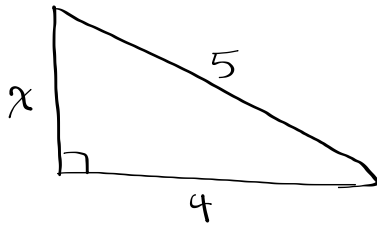
$$(6, 0) \text{ ; } (4, 0)$$

## Right Triangle


 $a \text{ \& } b \rightarrow \text{legs}$ 
 $c \rightarrow \text{Hypotenuse}$ 

$$a^2 + b^2 = c^2$$

Pythagorean Thrm



Find  $x$

$$x^2 + 4^2 = 5^2$$

$$x^2 + 16 = 25$$

$$x^2 + 16 - 25 = 0$$

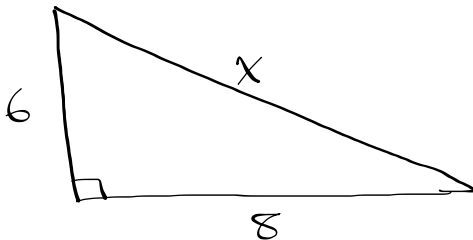
$$x^2 - 9 = 0$$

$$(x+3)(x-3) = 0$$

$$\downarrow$$
  
 ~~$x = -3$~~

$$\downarrow$$
  
 $x = 3$

Find  $x$



$$6^2 + 8^2 = x^2$$

$$36 + 64 = x^2$$

$$100 = x^2$$

$$x^2 = 100$$

$$x^2 - 100 = 0$$

$$(x-10)(x+10) = 0$$

$$\downarrow$$

$$x = 10$$

$$\downarrow$$

~~$x = -10$~~



The dimensions of a rectangular shape with area  $80 \text{ ft}^2$  are two cons. even integers.

Find its dimensions.

$$A = 80$$

$$LW = 80$$

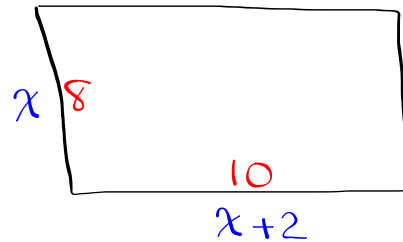
$$x(x+2) = 80$$

$$x^2 + 2x - 80 = 0$$

$$(x+10)(x-8) = 0$$

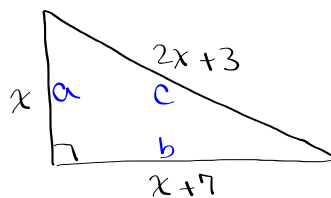
$$\swarrow$$
  
 ~~$x = -10$~~

$$\searrow$$
  
 $x = 8$



8 ft by 10 ft

Find all 3 sides:



Right Triangle

Pythagorean thrm

$$a^2 + b^2 = c^2$$

$$x^2 + (x+7)^2 = (2x+3)^2$$

$$x^2 + (x+7)(x+7) = (2x+3)(2x+3)$$

$$x^2 + x^2 + 7x + 7x + 49 = 4x^2 + 6x + 6x + 9$$

$$2x^2 + 14x + 49 = 4x^2 + 12x + 9$$

$$\underline{2x^2} + \underline{14x} + \underline{49} - \underline{4x^2} - \underline{12x} - \underline{9} = 0$$

$$-2x^2 + 2x + 40 = 0$$

Divide by -2

$$x^2 - x - 20 = 0$$

$$(x-5)(x+4) = 0$$

$$\swarrow$$
  
 $x = 5$

$$\searrow$$
  
 ~~$x = -4$~~

5, 12, and 13

Graph of  $y = x^2 - 36$  has two  $x$ -Ints.  
Find them.

$(6, 0)$  &  $(-6, 0)$

$$y = 0$$

$$x^2 - 36 = 0$$

$$(x+6)(x-6) = 0$$

$$x = -6$$

$$x = 6$$

Special Factoring: Trinomials

$$A^2 + 2AB + B^2 = (A + B)^2$$

$$x^2 + 50x + 625 = (x + 25)^2$$

$$9x^2 + 144x + 576 = (3x + 24)^2$$

This problem had a GCF.

$$= 9(x + 8)^2$$

$$49x^2 + 140xy + 100y^2 = (7x + 10y)^2$$

$$64x^2 + 400xy + 625y^2 = (8x + 25y)^2$$

$$A^2 - 2AB + B^2 = (A - B)^2$$

$$4x^2 - 12x + 9 = (2x - 3)^2$$

$$36x^2 - 60xy + 25y^2 = (6x - 5y)^2$$

Review exam 1 & exam 2

Project III due Monday

Lecture 6:00 - 7:45

Exam 7:50 - 9:30

Long Division, Exponential rules

Factoring, Similar word problems

operations with Polynomials

Graph & shade, Parallel & Perpendicular lines.